Al-Khāzinī's Complex Tables for Determining Lunar Crescent Visibility

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Keywords: Astronomy, astronomy of the Islamic Period, social history of astronomy, lunar crescent visibility, *zīj*, *Almagest*, 'Abd al-Raḥmān al-Khāzinī, Thābit ibn Qurra, Sanjar ibn Malikshāh, al-Battānī, Ibn Yunūs, Naṣīr al-Dīn al-Ṭūsī, *Sanjarī Zīj*, *Īlkhānī Zīj*, Maḥmūd ibn 'Umar, *Nāṣerī-Zīj*.

Abstract

Lunar crescent visibility is a multidisciplinary branch of astronomy from the Islamic period, which reflects the interrelationship between astronomy and Islam. Its importance endures today, since the Hijra Lunar Calendar is still organized according to the rules or criteria for lunar crescent visibility in different Muslim countries. In recent decades, several criteria for lunar crescent visibility from medieval Islamic astronomy have been studied by historians of astronomy, although many of these methods have not yet been investigated. Among the unstudied materials are two interesting tables in a valuable work entitled *al-Zīj al-mu'tabar al-sanjarī* by the celebrated astronomer and author of treatises on mechanics Abū al-Fatḥ 'Abd al-Raḥmān al-Khāzinī (fl. 474-525 A.H./ 1081-1131 A.D.).

In this article, I will analyze these two tables for determining lunar crescent visibility, included in al-Khāzinī's *Sanjarī Zīj*. The first one is simpler and is based on straightforward limits and computations. The second table, briefly introduced by Prof. Kennedy in 1956, allows the examination of lunar crescent visibility or invisibility on three levels. This more complex table is based on several astronomical and non-astronomical parameters,

without any explanation of the rules by which the array of numerical values were computed. Undoubtedly al-Khāzinī presented this criterion under the influence of Thābit ibn Qurra, but he endeavored to construct a new criterion which also showed the effect of human and atmospheric conditions on lunar crescent visibility. Al-Khāzinī distinguishes three levels of crescent visibility (general, moderate and acute), thus introducing a concept of frequency.

Introduction

In the Islamic lunar calendar, each month begins with the first observation of the lunar crescent above the western horizon. This fact led astronomers from the Islamic period to devise tables and criteria for determining lunar crescent visibility at the beginning of each lunar month. As modern research has shown, the two earliest criteria for lunar crescent visibility in Islamic astronomy probably appeared in the works of Ya'qūb ibn Ṭāriq and al-Khwārizmī, which were both heavily based on Indian astronomy.[1]

In later centuries, there were disagreements between Islamic astronomers on the subject of lunar crescent visibility and in fact it remains a complex problem even for modern astronomers, because it depends on different human, atmospheric and astronomical parameters. As far the texts from the Islamic period are concerned, the discrepancies in the methods can be easily recognized by studying and comparing the numerous criteria and tables in the surviving *zīj*es (astronomical handbooks with tables) and in books dealing with general astronomy (*hay'a*). Although in recent decades historians of astronomy have investigated some of these tables and criteria, many of them remain unexamined.[2]

[1] For Ya'qūb ibn Ṭāriq's criterion see Kennedy, 1968, pp. 126-132. The presence of a table for lunar crescent visibility in al-Khwārizmī's original *zīj* cannot be ascertained from the commentaries by Ibn al-Muthannā and Ibn Masrūr. However, a table attributed to al-Khwārizmī can be found in various sources (King, 1987, 189-192). This table can be shown to be based on the Indian visibility criterion with obliquity of the ecliptic 23° 51' and geographical latitude 33°. The different table in al-Majrīṭī's recension was studied by Kennedy and Janjanian, 1965, pp.73-78; reprinted in Kennedy et al., 1983, pp. 151-156 and by King, 1987, pp. 192-197. After a systematic analysis, Hogendijk (1988 (2), pp. 32-35) concluded that the table was based on an Indian visibility criterion and either obliquity 23° 35' and latitude 41° 35' or obliquity 23° 51' and latitude 41° 10'. See also Van Dalen, 1996, p. 205.

[2] See King, "Ru'yat al-Hilāl" [= lunar crescent visibility] in the *Encyclopedia of Islam* (EI^2); see Kennedy, 1956, for significant *zīj*es containing tables on lunar crescent visibility. For a list of previous investigations of lunar crescent visibility in Islamic astronomy, see King, 1993, II,

Among the unstudied material, we find two interesting tables in a valuable work entitled *al-Zīj al-mu'tabar al-sanjarī* by the celebrated astronomer and author of treatises on mechanics Abū al-Fatḥ 'Abd al-Raḥmān al-Khāzinī (fl. 474-525 A.H./ 1081-1131 A.D.), who lived in Merv (an important city in ancient Khurāsān of Iran, now Mary in Turkmenistan).

Al-Khāzinī dedicated his work to the sultan Sanjar ibn Malikshāh (sultan of the Saljuqid empire, reign: 512-552 A.H./ 1118-1157 A.D.). The *Sanjarī Zīj* is particularly important due to the various innovations (including a new theory on lunar crescent visibility) that were listed by the author to show its superiority over previous *zīj*es. Moreover, in the introduction he states that he had made observations during a period of thirty-five years in order to compile this *zīj*.[3]

According to Kennedy's research, al-Khāzinī was among twenty-two individuals or groups of astronomers from the Islamic period who made independent observations.[4] In spite of its importance, the text of the *Sanjarī Zīj* as a whole still awaits detailed study.

In his *Survey of Islamic astronomical tables* published in 1956, Prof. E.S. Kennedy stated that the *Sanjarī Zīj* includes the most comprehensive

p. 219, or III, p. 166, on lunar crescent visibility and the regulation of the Islamic calendar. For analyses of the underlying parameters of some lunar crescent visibility tables, see Hogendijk, 1988 (1), pp. 95-104 and 1988 (2), pp. 29-44.

[3] For biographical information on al-Khāzinī, see the article "al-Khāzinī" by Robert E. Hall in *DSB*, vol. 7, pp. 335-351, and for a historical source see Baihaqī, p. 131. There are three main manuscripts of the *Sanjarī Zīj*: Vatican, Arab 761 (abbreviated as Vat), British Museum, Or. 6669 (BM) and Sh. Moṭahhari (formerly Sepahsālār) Library, No. 682, Tehran (MT). Al-Khāzinī also compiled an abridgement of his *zīj* (*Wajīz al-Zīj*) in 525 A.H/1131A.D. (extant in the Ḥamidiyye Library, no. 589, Istanbul, abbreviated as HM), See Kennedy, 1956, p. 129. From al-Khāzinī's reference to his thirty-five years of observational work in the introduction of the *Sanjarī Zīj* (Vat. fols. 16v-17r) and his table of star positions (fol. 191v) dated 509 A.H./1115 A.D., I have concluded that he was an active astronomer at least from 474 A.H./1081 A.D. This conclusion is supported by the citations of some historical sources that tell us that al-Khāzinī was among the astronomers at the court of Malikshāh, who revised the Persian solar calendar in 468 or 471 A.H./1076 or 1079 A.D., which led to the Malikī or Jalālī Calendar. See Sédillot, vol. 52, pp. 309-310 (for its French translation see vol. 53, p. 27) and Sayili, p. 165. Further, 'Alī Shāh-e Bukhārī in the '*Umdat al-Īlkhāniyya* (ms. 781 Paris, fol. 6r) mentions that al-Khāzinī established the Malikī Calendar in Merv because of the order of Malikshāh.

[4] Kennedy, 1956, p. 169.

discourse on lunar crescent visibility among all *zīj*es he had seen.[5] He also presented a summary of this *zīj*, with a short explanation of the astronomical basis of the lunar crescent visibility tables of al-Khāzinī and Thābit ibn Qurra.[6]

In 1965, Kennedy analyzed Thābit ibn Qurra's table, which is only extant in the *Sanjarī Zīj,* near al-Khāzinī's own table.[7] In this article, I will analyze two of al-Khāzinī's tables for determining lunar crescent visibility in the *Sanjarī Zīj*. With the second table, it is possible to examine the condition of lunar crescent visibility or invisibility on three levels; it can therefore be considered as a developed, more sophisticated version of Thābit's criterion.

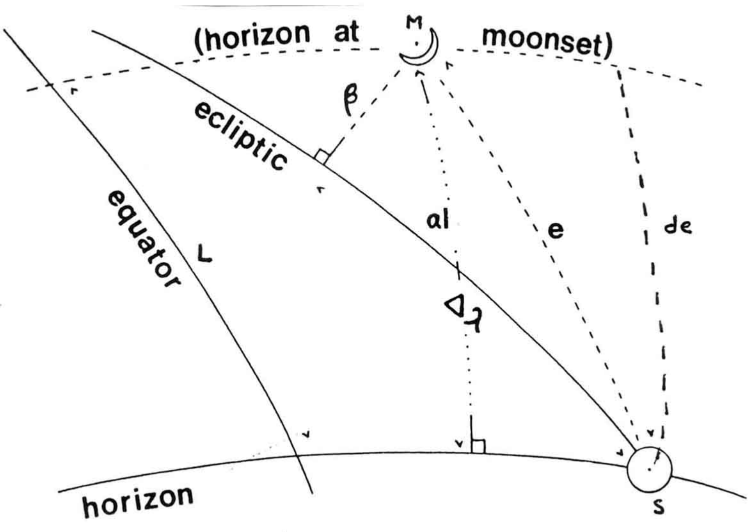

Fig. 1: The various arcs between the sun and the moon above the observer's western horizon (adopted from King 1991, p. 241).

[5] Undoubtedly, the *Sanjarī Zīj* is among the most extensive examples on many topics including lunar crescent visibility. However, the recent literature on the problem of lunar crescent visibility has not pointed to any comparably wide treatment in other *zīj*es.

[6] Kennedy, 1956, p. 160.

[7] Kennedy, 1960, pp. 71-74; reprinted in Kennedy et al., 1983, pp. 140-143. See also Morelon, pp. 113-116, 256-259. Thābit also wrote a separate treatise on lunar crescent visibility which is similar to Thābit's material in the *Sanjarī Zīj*, see Morelon, pp. 93-112, 230-255.

The astronomical basis of al-Khāzinī's criteria

Fig. 1 shows various relevant arcs in relation to the moon, *M*, and the sun, *S*, above the observer's western horizon. In this figure the symbols are as follows: *e* is the angular separation between the centers of the moon and the sun, *de* the depression of the sun at moonset, *L* the time lag between sunset and moonset computed as degrees on the celestial equator ($4^m \equiv 1^\circ$), $\Delta\lambda$ the arc between the sun and the perpendicular projection of the moon on the ecliptic, and *al* the lunar altitude.

In the astronomical tradition of the Islamic period, various critical values for some of these arcs were used in the construction of different visibility criteria. In al-Khāzinī's criteria arc *e* is called the arc of light, *qaws al-nūr*; *de* the arc of the solar depression, *qaws inḥiṭāṭ al-shams*; *al* the arc of altitude, *qaws al-irtifā'*; and *L* the arc of tarrying *qaws al-makth*. Moreover, he takes account of V_m, the moon's *buht* (*buht al-qamar*), which indicates the daily angular velocity of the moon[8] (which varies approximately between 11.834º at apogee and 14.716º at perigee)[9], and the lunar anomaly, *khāṣṣat al-qamar*. These two latter values can be computed from each other, and they are directly related to the lunar distance from the earth. This distance has a noticeable effect on the visibility of the lunar crescent, but was rarely considered by Islamic astronomers. Only three other models for crescent visibility are known before al-Khāzinī's time: the models of Thābit ibn Qurra, al-Battānī and Ibn Yūnus, which also used the effect of the lunar distance from the earth although in different ways.[10] It seems that al-Khāzinī was unaware of Ibn Yūnus' work.

[8] The concept of *buht* originated from Indian astronomy and was apparently transferred to the astronomy of the Islamic period through the *Zīj* of al-Khwārizmī; see Neugebauer, p. 57. For the different minimal and maximal values of *buht* in Indian astronomy, see "History of mathematical Indian astronomy" by Pingree, in *DSB* vol. 15, pp. 539, 541, 545. For another historical explanation, see al- Bīrūnī, *al-Qānūn*, pp. 875-877.

[9] These extreme values are for an undisturbed orbit, the true daily lunar velocity with regard to perturbations changes to 15.301º in the case of maximum and 11.799º in the case of minimum. See Maeyama, pp. 269-283.

[10] The astronomers from the Islamic period normally modified the critical values of visibility according to lunar anomaly (like Thābit and al-Battānī) or lunar angular velocity (like Ibn Yūnus) to consider lunar distance. Both parameters are derived from the lunar distance, but it should be kept in mind that they never directly used the linear distance of the moon as a parameter in modifying the values. For Thābit's criterion see Kennedy, 1960. For Ibn Yūnus' criterion see King, 1988, pp. 155-168, reprinted in King, 1993, III. For al-Battānī's criterion see Bruin, pp. 331-358.

Al-Khāzinī's text in the *Sanjarī Zīj*[11]

I now present my English translations of the texts describing al-Khāzinī's criteria from ms. Arab. 761 (Vatican library), which is the most complete and reliable manuscript. My explanatory additions to the translation are provided in angular brackets < >, and my comments are shown in parentheses ( ). The sentences which are related to the process of computation have mostly been translated in the present tense. Al-Khāzinī frequently uses the Arabic word *faṣl* (section) to separate different subjects; I have omitted this from my translations.

Ninth essay (*maqāla*), second chapter (*bāb*): Report on those who relied on single simple arcs

> There are four single arcs which are used to determine crescent visibility. One of them is the "arc of light", the second "the arc of tarrying", the third "the arc of depression", and the fourth "the arc of altitude". A group of "people of the Book" < i.e. Christians and Jews > said that, <if the arc of altitude (?) is comprised >[12] between 10° and 12° or more, the moon is within the limits of visibility and it may be seen. The al-Hind wa'l-Sind[13] <astronomers> and Muḥammad ibn Mūsa al-Khwārizmī said that if the arc of tarrying is 12° or greater, the <lunar> crescent is visible; otherwise it is not <visible>. Aḥmad ibn 'Abd Allāh Ḥabash al-Ḥāsib said that when the solar depression at moonset is 10° or more, <the lunar crescent> is visible and, otherwise, it is not <visible>.Al-Sa'īdī transmits from him that when <the solar depression is> 8° 2/3 or more, <the lunar crescent> will be seen, and if it is less than that, it will not. Kūshyār said: "In my opinion", if the arc of light is 10°, the arc of tarrying is

[11] Fols. 85v-91r (ms. Vat.) contain the whole discussion on lunar crescent visibility but the second, third and sixth chapters just appear on fols. 87r-88r, 89v-90v.

[12] It was impossible to recover this passage from mss., BM and MT. Here, the arc of altitude could be replaced by the "arc of vision" used in *Alfonsine Tables* and derived from Maimonides (see Chabás and Goldstein, p. 199-200). The problem, however, is that al-Khāzinī did not discuss the concept of "arc of vision" in his *Zīj*, and the origin of Maimonides' criterion is unclear.

[13] Prof. Samsó supposed that the Arabic expression *al-Hind wa'l Ṣīn,* followed by the mention of al-Khwārizmī may be an Arabic corruption of *al-Hind wa'l-Sind* = the *Sindhind*, the astronomical school based on Indo-Iranian methods adopted by al-Khwārizmī.

8º, the arc of apparent altitude is 6º and the arc of <solar> depression is comprised between 8º and 7;30º, <namely> if <all> these arcs are in their <maximum> limits <or> more, then the crescent is in the limit of visibility, and it may be observed. If they are less <than those limits>, then <the lunar crescent> is not <visible>". <Kūshyār also> said that if the sum of the arcs of light and depression is 18º or more, <the lunar crescent> will be seen and if two of them (i.e., arcs) certify the <crescent visibility> then its visibility may be predicted.

Third chapter: On al-Khāzinī's correction <based> on four single arcs

The<values of> the four mentioned arcs must be corrected according to the position of the moon on <its> epicycle, because when the moon is at the apogee of the epicycle their computed values will be greater than when the moon is at the perigee of the epicycle. The first of them is "the arc of light" <whose minimal value> is comprised between 10º and 12º. The second is "the arc of tarrying" <whose minimal value> is comprised between 7;30º and 9º <and> the fourth is "<the arc of> apparent altitude" <whose minimal value> is between 6 2/3º and 8º approximately.[14] When we want to modify them, we take, from the table of eclipses, the "minutes of correction" (*daqā'iq al-taqwīm*) corresponding to the lunar anomaly. We multiply them by the difference <between the extreme values> of each of them <i.e., each arc> and divide the amount obtained by 60. Then we subtract the result from the greater one (i.e., the upper extreme of each arc). Then the desired arc remains. We may also calculate it using the moon's *buht*. The method is <as follows>: we take it (i.e., the moon's *buht*) in minutes<of arc> and subtract 727 from it. Then we multiply the remainder by the difference <between the extreme values of each arc> and the result is divided by 138, and we then subtract the quotient from the greater one (i.e., the upper

[14] There is a difference between extreme tabular values of the arcs (ms. Vat., fol. 88r) and the values given in the text; for the arc of tarrying, the table gives: 8;20º ≤ L≤ 10º. Moreover, here, due to a mistake, there is no mention of the arc of depression as the third arc, but the table includes extreme values for the arc of depression. For its translation, see Table 1.

extreme values of each arc). The result will be the desired arc. We keep each of them (i.e., the arcs) separate. To <determine> the lunar crescent visibility from them, we return to the arcs obtained according to the positions of the two luminaries (i.e., the moon and the sun) for the thirtieth evening <of the lunar month> and compare them with the kept <arcs>. If they are the same as or greater than them (i.e., the kept arcs), the crescent lies in the limit of visibility, and it may be observed; if they are less <than those>, then <the crescent> cannot be observed. If some <arcs> indicate visibility <and some otherwise>, the judgment has to be made according to the majority. In this field one should rely on a composite criterion based on the arcs of light and depression, dismissing the other <arcs>. Because when the arc of light exceeds 24°, the crescent will be visible <in daylight> before sunset, even if the arcs of tarrying, depression and altitude do not indicate visibility in some places and localities.

Sixth chapter: Mention of al-Khāzinī's considerations regarding <lunar crescent> visibility

This chapter refers to the computation of the total "<arc of> solar depression" as a function of the "arc of light". Once it has been calculated, it is called the "corrected arc of visibility". We say: we compute the ecliptic longitude of the two luminaries (i.e., the sun and the moon) at the moment of moonset, for the thirtieth evening of the Arabic month. As quoted before, we obtain from their corrected ecliptic longitudes, the corresponding arcs of light and depression and we keep them. In order to determine the total limits from the table, we enter the anomaly of the moon <or the *buht* of the moon> in one of the three tables of the limits of visibility (i.e., the triad table) given by al-Khāzinī, whichever is more appropriate for the season <of the year> and the locality under consideration, according to one's experience of the subject. We take the corresponding two arcs as the first and the second limits and we keep each one separate. Then we subtract the first limit from the second one, we call the result the "arc of correction" (*qaws al-ta'dīl*) and we also keep it aside. In order to establish <visibility>, we consider first the "found arc of light" (i.e., the computed arc of light). If it is less than the arc of the first limit, then we do not expect to observe the lunar crescent,

because it is still under the rays <of the sun> (i.e., the moon is on the wane), and it cannot reach the limit of visibility. And if it is equal to or greater than the second limit, it has emerged from the <sun's> rays, it can be observed in daylight, before sunset, and does not require any other consideration. In order to correct the "arc of visibility", (i.e., the "computed arc of light") <we should consider> if the "arc of visibility" is greater than the first limit and less than the second one, and is therefore in a doubtful range. <It may be> on the side of necessary visibility <or on the side> in which visibility is impossible. So we need more attention and work. The method is <as follows>: we first obtain the "arc of total visibility", considering the distances (*ab'ād*) <from the apogee>, by subtracting the first limit, which resulted from the table, from the "found arc of light" (i.e., the computed arc of light), and obtain the remainder (*al-faḍla*). This is multiplied by the first limit, and the result is divided by the <arc of> correction (*ta'dīl*). Then we always subtract the quotient from the first limit and the remainder will be the "arc of visibility". And if we want <to compute in another way:> we subtract the remainder from the "kept correction" and the remainder is the "portion (*ḥiṣṣa*) of the arc of visibility". Then we multiply it by the first limit and we divide the result by the "<arc of> correction": the result is the "arc of total visibility". In order to check <visibility> again, we return to the "<arc of > the solar depression". If it is equal to or greater than that of the "arc of total visibility", the crescent can be seen at that place, and if it is less, the crescent will not be observed. This is the basic <method> for <determining lunar crescent visibility at> a specific place, as proved by experience<s> in one of the three <tables>. If we want <to establish> a universal method and <arrive at> a rule <valid> for all places, we apply the aforementioned procedure to each one of the three tables. Then after finishing it, if the result indicates visibility, we say that the visibility is "general" (*shā'i'a*) if the positive result is obtained from the first table. If it has been obtained from the second, but not the first, <we conclude that> the moderate (*mu'tadila*) and acute (*ḥādda*) visions will see it (i.e., the crescent), if the atmosphere is clear but <the visibility> will not be general (*shā'i'a*). And if the positive result has been obtained from the third table, but not from the first two <we conclude that only > people with acute sight will see <the crescent> on rare occasions if the atmosphere is clear enough. And when <even> the third <table>

does not indicate <visibility>, there is agreement that the crescent will not be seen anywhere due to the lack of light in its body (i.e., surface).

Commentary

As the text shows, al-Khāzinī presents two models for determining lunar crescent visibility. The first one is simpler, and it can predict the condition on the basis of a computation of the four arcs *e*, *L*, *de*, and *al*. This criterion considers two extremes for each arc, and the values should be modified according to the lunar distance from the earth. The lunar distance enters the criteria through the lunar velocity V_m (in minutes of arc) or the lunar anomaly *a* (in degrees). Table 1 shows the extreme values for each arc.[15]

	P_1	P_2
e	10;00º	12;00º
L	8; 20	10;00
de	7;30	9;00
al	6;40	8;00

Table 1: Al-Khāzinī's critical values for lunar visibility at perigee (P_1) and apogee (P_2), from ms. Vat., fol. 88r.

If the moon is at the perigee, the critical value for its visibility is the smallest one (i.e., P_1) and at the apogee the largest one (i.e., P_2). For lunar distances between perigee and apogee, the critical values for visibility should be computed by one of the following equivalent formulas:

$$K = P_2 - (f(a).\, \Delta\, m / 60) \qquad (1)$$

[15] The ms. BM, presents the following extremes: 10º ≤ *e* ≤ 12º, 8;20º ≤ *L* ≤ 10º, 7;30º ≤ *de* ≤9º, 6;40 º ≤ *al* ≤ 8º. Both mss., MT, fol. 18r and HM, fol. 30v, give the extremes as: 10º ≤ *e* ≤ 12º, 8º ≤ *L* ≤ 12º, 8º ≤ *de* ≤10º, 6 º ≤ *al* ≤ 8º, but in HM there is a marginal quotation from the original copy of the *Sanjarī Zīj*, which presents the same extremes as BM, but with a difference in the lower extreme of altitude as: 7;40 º ≤ *al* ≤ 8º.

Or: $$K = P_2 - (\Delta m.(V_m - 727)/138) \qquad (2)$$

In these formulas $\Delta m = P_2 - P_1$ is computed for each one of the four arcs separately, and the modified values K for the four arcs are found by means of interpolation. In Formula 1, $f(a)$ is a trigonometric interpolation coefficient corresponding to the anomaly a, which is taken from a table in Book VI of the *Almagest*[16] (Table 2). In formula 2, 727 is the minimum value of V_m and 138 the difference between its maximum and minimum, both expressed in minutes of arc.

Table of Correction		
anomaly	anomaly	sixtieths $f(a)$
6°	354°	0' 21"
12	348	0 42
18	342	1 42
24	336	2 42
30	330	4 01
36	324	5 21
42	318	7 18
48	312	9 15
54	306	11 37
60	300	14 00
66	294	16 48
72	288	19 36
78	282	22 36
84	276	25 36
90	270	28 42
96	264	31 48
102	258	34 54

[16] In the *Sanjarī Zīj* (Vat., fol. 141v), al-Khāzinī uses the same values as in Table VI 8 of the *Almagest* (see Table 2). Of course, Ptolemy, p. 308 applies this table to modify the obscurity of eclipses on the basis of lunar anomaly, but here al-Khāzinī uses it in lunar crescent visibility. Al-Battānī, in his *Zīj al-Ṣābī*, before al-Khāzinī, also applies this table to modify the arcs of lunar visibility. See Nallino, for its Latin translation part I, p. 87, for the Table, part II, p.89, 231, for its Arabic edition, part III, p. 131. For some explanations of al-Battānī's criterion, see Bruin, pp. 331-358.

108	252	38 00
114	246	41 00
120	240	44 00
126	234	46 45
132	228	49 30
138	222	51 39
144	216	53 48
150	210	55 32
156	204	57 15
162	198	58 18
168	192	59 21
174	186	59 41
180	180	60 00

Table 2: From the *Almagest* Book VI 8, p. 308.

Al-Khāzinī states that if some arcs indicate visibility and some others do not, the visibility should be decided on the basis of the arcs e and de, because when $e \geq 24^{\circ}$, the crescent is visible in daylight, and there is no need to compute the other arcs as well.[17] This shows that al-Khāzinī considered L and al as supplementary arcs for the determination of visibility. Formulas 1 and 2 imply that al-Khāzinī considered a mathematical relation between V_m and $f\,(a)$. This relation is easily obtained, if one compares formulas 1 and 2 as follows:

$$f\,(a) = (V_m - 727)/2.3 \qquad (3)$$

[17] There is no doubt that al-Khāzinī considered that a reliable prediction should be based on the four arcs. Thus, if all four arcs are greater than the critical values, the crescent will definitely be visible. On the other hand, it can be shown that normally $al < e$, so that it is possible having $e > 10^{\circ}$ and $de > 7;30^{\circ}$ but $al < 6;40^{\circ}$ and $L < 8;20^{\circ}$. Of course, in these cases the values of al and L cannot be much lower than the critical (tabular) values particularly in mid-geographical latitudes.

It should be noticed that in the reverse situation (where both al and L are greater than their critical values and e, de are smaller), the lunar visibility is not reliable. The best naked eye observations show that the minimum obtained limit of e is near 10°. Since these cases could cause problems for calendar organizers in the prediction of crescent visibility, it seems that al-Khāzinī tried to solve the problem by emphasizing the two mentioned arcs (e and de). However, this prediction (based on two arcs) bears a degree of uncertainty compared with a four-arc prediction.

The second criterion consists of a computational procedure appended by a table which has different extreme values in the two groups of mss. Vat., BM and MT, HM (Tables 3 and 4).[18] Both tables deal with visibility on three levels: general (*shā'i'*, i.e., easily visible), moderate (*mu'tadil*, i.e., moderately visible) and acute (*nādir*, i.e., the lunar visibility is acute and it is rarely visible). Obviously one first checks the visibility at the "general" level and if the crescent is not observable there, it should be checked respectively at the two other levels.

The variables V_m and a reappear as corresponding parameters in the second criterion.[19] The values of V_m in Tables 3 and 4, begin from 12;06º, thus 727' in formula 2, should be changed to 726' (=12;06º) and the greatest value of V_m reaches 14;27º (= 867') therefore the difference between minimum and maximum is 141' and this value is applied instead of 138' in formula 2.[20] Because of these modifications, formula 3 (with regard to the values of V_m in Tables 3 and 4) should be changed as follows:

$$f(a) = (V_m - 726)/2.35 \qquad (4)$$

According to formula 4 the recomputed values of V_m deviate over 8' in some cases (see Table 3).

The deviations of the values recomputed from tabular values cannot be considered as normal errors in computation. The main question is whether formula 4 basically yields correct values for V_m (in a limit of a minute of arc) or not. Since Tables 3 and 4 are not specialized enough to show the relation between a and V_m, one cannot have confidence in the values given. There is a table in ms. Vat. fol. 135 v entitled: "Table of the lunar path (velocity) and

[18] There are some negligible differences between the values of each group of manuscripts, which may be due to copyists' errors.

[19] The values of Thābit's table correspond to a and, because al-Khāzinī's table is next to Thābit's, each value of V_m in al-Khāzinī's table may correspond to a value of a as well (See the plate in the appendix). Indeed the recomputed values of V_m by means of formulas 4 and 5 and the values of $f(a)$ confirm this conjecture.

[20] It is very likely that the range: $12;6^\circ \leq V_m \leq 14;27^\circ$ in al-Khāzinī's table is based on Ptolemy's method for computing lunar velocity according to his first lunar model in the *Almagest* (p. 282). This lunar model leads to the lunar true daily motion with extreme values: $12;7^\circ \leq V_m \leq 14;26^\circ$, which are near al-Khāzinī's values in Tables 3 and 4; see also Goldstein, pp. 5,7. It seems that al-Khāzinī extends the extremes of V_m in Tables 3 and 4 to cover some rare cases in lunar visibility.

its diameter and the diameter of the shadow" (*Jadwal masīr al-qamar wa quṭri-hi wa quṭr al-ẓill*). This table tabulates the values of a corresponding to V_m, the angular diameter of the moon and the angular diameter of the earth's shadow respectively.

In this table, the values for V_m are given with a precision of a second of arc, and so obviously the table is more accurate than Tables 3 and 4, rounded to the nearest minute. I have just dealt with the two columns concerning a and V_m (see Table 5). First of all we should mention that the extreme values of V_m in it are 12;6,7° and 14;28,35° compared with Tables 4 and 5 with values between 12;6° and 14;27°.

The values of V_m in Table 5 are accurate to less than one minute of arc by formula 4, which (unlike Tables 3 and 4) does not produce differences in accuracy.

But the problem is that the extreme values of V_m in Table 5 differ slightly from those appearing in Tables 3 and 4, so formula 4 needs the following slight modification:

$$f(a) = (V_m - 726.116)/2.374 \qquad (5)$$

The latter equation agrees better with the values given in Table 5. It is clear that the differences between the recomputed and the original values do not reach a minute of arc in any of the cases (see Table 5). Since the increment of anomaly in Table 5 is 4°, just the values of $f(a)$ of the arguments in bold are accessible from Table 2. In the *Almagest*, Table V 18 (col. 7) gives the values of $f(a)$ for the increment of 4°.[21] Thus, recalculations have been done for the rest of arguments based on the latter Table. Finally it can be said that formula 4 is correct and that the values of V_m corresponding to a in Tables 3 and 4, because of the mistakes, do not agree exactly with the corresponding values of a as computation shows.

[21] The interpolation function in the *Almagest* V 18 (col. 7) is the same as the one in *Almagest* VI. 8 (col. 3). The difference goes back to the fact that the arguments of V 18 are between 0-90° while those of VI 8 are between 0-180°. Thus if each argument of V 18 is multiplied by 2, this gives its corresponding value in VI 8, if it is a multiple of 6°. In fact multiplication of the arguments of V 18 by 2 gives us a table of argument with 4° interval. For Table V 18 see Ptolemy, *Almagest* p. 265; al-Khāzinī (Vat., fol. 138 v) presents Table V18, with the same tabular values.

a	V_m	General (easily visible)		moderate (moderately visible)		acute (rarely visible)	
		e_1	e_2	e_1	e_2	e_1	e_2
0º	12;06º(0)	11;50º(0)	26;55º(0)	10;45º(0)	25;25º(0)	9;40º(0)	23;55º(0)
6	12;07(0)	11;49(0)	26;55(+1)	10;45(+1)	25;25(+1)	9;40 (+1)	23;55(+1)
12	12;09(+1)	11;48(0)	26;54(+2)	10;44(+1)	25;24(+2)	9;39 (+1)	23;54(+2)
18	12;12(+2)	11;47(+1)	26;52(+3)	10;42(+1)	25;22(+3)	9;38 (+2)	23;53(+3)
24	12;15(+3)	11;45(+2)	26;50(+4)	10;40(+1)	25;20(+3)	9;37 (+3)	23;52(+4)
30	12;18(+3)	11;42(+1)	26;47(+4)	10;38(+1)	25;18(+4)	9;34(+2)	23;49(+3)
36	12;22(+3)	11;40(+2)	26;44(+4)	10;36(+2)	25;15(+4)	9;32(+2)	23;47(+4)
42	12;26(+3)	11;37(+3)	26;41(+5)	10;33(+2)	25;12(+4)	9;30(+3)	23;45(+5)
48	12;30(+2)	11;34(+3)	26;36(+3)	10;29(+2)	25;08(+4)	9;27(+3)	23;42(+5)
54	12;34(+1)	11;30(+3)	26;31(+2)	10;25(+1)	25;05(+4)	9;23(+2)	23;39(+4)
60	12;38(-1)	11;25(+2)	26;26(0)	10;21(+1)	25;01(+2)	9;19(+1)	23;34(+2)
66	12;42(-3)	11;20(+1)	26;23(0)	10;16(0)	24;58(+2)	9;15(+1)	23;31(+1)
72	12;47(-5)	11;15(+1)	26;19(0)	10;11(-1)	24;53(+1)	9;10(0)	23;28(+1)
78	12;54(-5)	11;10(+2)	26;14(+1)	10;05(-2)	24;48(+1)	9;06(0)	23;22(-1)
84	13;02(-4)	11;04(+2)	26;09(+3)	10;00(-2)	24;42(+2)	9;01(0)	23;18(+1)
90	13;10(-3)	10;58(+2)	26;03(+4)	9;56(0)	24;36(+2)	8;57(+1)	23;14(+2)
96	13;18(-3)	10;53(+3)	25;58(+5)	9;52(+1)	24;30(+2)	8;52(+1)	23;10(+4)
102	13;24(-4)	10;47(+2)	25;52(+4)	9;47(+1)	24;26(+2)	8;48(+1)	23;05(+2)
108	13;30(-5)	10;41(+1)	25;47(+3)	9;42(+1)	24;22(+2)	8;43(0)	23;01(+1)
114	13;37(-5)	10;35(+0)	25;42(+4)	9;37(0)	24;18(+3)	8;39(+1)	22;58(+3)
120	13;43(-6)	10;29(-1)	25;37(+3)	9;32(0)	24;14(+3)	8;34(0)	22;54(+2)
126	13;50(-6)	10;24(-1)	25;32(+4)	9;27(-1)	24;09(+3)	8;30(0)	22;49(+1)
132	13;56(-6)	10;19(-1)	25;27(+3)	9;24(+1)	24;05(+3)	8;26(0)	22;46(+2)
138	14;01(-6)	10;15(-1)	25;22(+2)	9;20(0)	24;02(+4)	8;22(-1)	22;42(+1)
144	14;05(-7)	10;12(-1)	25;17(0)	9;16(-1)	23;58(+2)	8;19(-2)	22;38(-1)
150	14;09(-8)	10;09(-1)	25;13(-1)	9;12(-2)	23;53(0)	8;16(-2)	22;35(-2)
156	14;13(-8)	10;08(+1)	25;10(-1)	9;10(-2)	23;50(-1)	8;14(-2)	22;32(-3)
162	14;17(-6)	10;05(0)	25;07(-1)	9;08(-1)	23;47(-1)	8;13(-1)	22;30(-2)
168	14;21(-4)	10;04(+1)	25;05(0)	9;07(+1)	23;44(-1)	8;12(0)	22;28(-1)
174	14;25(-1)	10;02(+1)	25;02(0)	9;06(0)	23;42(0)	8;11(0)	22;26(0)
180	14;27(0)	10;00(0)	25;00(0)	9;05(0)	23;40(0)	8;10(0)	22;25(0)

Table 3: The parameters of Al-Khāzinī's second criterion in three levels from ms. Vat., fol. 143r.

V_m	general (easily visible)		moderate (moderately visible)		acute (rarely visible)	
	e_1	e_2	e_1	e_2	e_1	e_2
12;06º	11;50º(0)	26;00º(0)	10;40º(0)	23;28º(0)	9;30º(0)	20;55º(0)
12;07	11;50(0)	26;00(+1)	10;40(0)	23;28(0)	9;30(0)	20;54(0)
12;09	11;50(+1)	25;59(+2)	10;40(+1)	23;27(+1)	9;30(+1)	20;53(0)
12;12	11;49(+1)	25;58(+3)	10;39(+1)	23;26(+3)	9;29(+1)	20;52(+2)
12;15	11;48(+1)	25;57(+4)	10;38(+1)	23;24(+3)	9;28(+1)	20;50(+2)
12;18	11;47(+1)	25;55(+4)	10;37(+1)	23;22(+3)	9;27(+1)	20;49(+3)
12;22	11;46(+2)	25;53(+5)	10;36(+2)	23;20(+4)	9;26(+1)	20;47(+4)
12;26	11;45(+2)	25;50(+5)	10;35(+2)	23;18(+4)	9;25(+2)	20;45(+5)
12;30	11;44(+2)	25;47(+4)	10;34(+2)	23;15(+4)	9;24(+2)	20;42(+5)
12;34	11;42(+1)	25;44(+3)	10;32(+1)	23;11(+2)	9;22(+1)	20;38(+3)
12;38	11;40(0)	25;40(+1)	10;30(0)	23;07(+1)	9;20(0)	20;34(+2)
12;42	11;38(-1)	25;36(-1)	10;28(-1)	23;03(-1)	9;18(-1)	20;29(-1)
12;47	11;36(-2)	25;32(-3)	10;26(-2)	22;59(-2)	9;16(-1)	20;25(-2)
12;54	11;35(-1)	25;28(-2)	10;25(-1)	22;54(-3)	9;15(0)	20;20(-2)
13;02	11;33(0)	25;24(-1)	10;23(0)	22;49(-3)	9;13(0)	20;16(-1)
13;10	11;31(0)	25;20(0)	10;21(0)	22;44(-2)	9;11(0)	20;12(+1)
13;18	11;29(0)	25;16(+1)	10;19(+1)	22;39(-2)	9;09(+1)	20;07(+1)
13;24	11;27(0)	25;12(0)	10;16(-1)	22;35(-3)	9;06(-1)	20;03(0)
13;30	11;25(-1)	25;07(-2)	10;14(-1)	22;31(-4)	9;04(-1)	19;58(-1)
13;37	11;22(-2)	25;02(-3)	10;12(-1)	22;28(-2)	9;02(-1)	19;54(-1)
13;43	11;20(-2)	24;58(-4)	10;10(-2)	22;23(-4)	9;00(-2)	19;49(-2)
13;50	11;18(-2)	24;54(-4)	10;08(-2)	22;19(-4)	8;58(-2)	19;45(-2)
13;56	11;16(-3)	24;51(-4)	10;07(-1)	22;15(-5)	8;57(-1)	19;41(-2)
14;01	11;15(-2)	24;48(-4)	10;05(-2)	22;11(-6)	8;55(-2)	19;38(-3)
14;05	11;14(-3)	24;45(-5)	10;04(-2)	22;07(-8)	8;54(-2)	19;34(-4)
14;09	11;13(-3)	24;42(-6)	10;03(-2)	22;04(-9)	8;53(-2)	19;31(-5)
14;13	11;12(-3)	24;40(-6)	10;02(-2)	22;03(-7)	8;52(-2)	19;29(-5)
14;17	11;11(-2)	24;39(-4)	10;01(-2)	22;02(-6)	8;51(-2)	19;27(-4)
14;21	11;11(-1)	24;38(-3)	10;01(-1)	22;01(-4)	8;51(-1)	19;26(-3)
14;25	11;10(-1)	24;37(0)	10;01(0)	22;01(-1)	8;51(0)	19;25(-1)
14;27	11;10(0)	24;36(0)	10;00(0)	22;01(0)	8;50(0)	19;25(0)

Table 4: The parameters of Al-Khāzinī's second criterion from ms. MT, fol. 49v.

a	V_m	Rec.	a	V_m	Rec.
0°	12;6,7°	0	92	13;16,27°	-0;0,15
4	12;6,8	-0;0,32	**96**	13;21,22	-0;0,15
8	12;6,13	-0;0,31	100	13;26,13	-0;0,18
12	12;7,45	-0;0, 2	104	13;31,4	-0;0,21
16	12;8,47	-0;0,35	**108**	13;36,5	-0;0,15
20	12;10,24	-0;0,33	112	13;40,55	-0;0,10
24	12;12,1	-0;0,31	116	13;45,41	-0;0,9
28	12;14,7	-0;0,30	**120**	13;50,27	- 0;0,7
32	12;16,13	-0;0,30	124	13;54,47	-0;0,8
36	12;18,50	+ 0;0,1	128	13;59,2	-0;0,14
40	12;21,27	-0;0,27	**132**	14;3,0	-0;0,38
44	12;24,21	-0;0,38	136	14;6,48	-0;0,12
48	12;27,35	-0;0,30	140	14;10,23	-0;0,3
52	12;31,43	-0;0,7	**144**	14;13,43	-0;0,7
56	12;35,10	-0;0,26	148	14;16,33	-0;0,1
60	12;39,3	-0;0,18	152	14;19,17	-0;0,1
64	12;43,25	-0;0,22	**156**	14;21,24	-0;0,38
68	12;47,11	-0;0,24	160	14;23,31	-0;0,10
72	12;52,18	-0;0,21	164	14;25,17	-0;0,4
76	12;57,4	-0;0,20	**168**	14;26,12	-0;0,49
80	13;1,50	-0;0,19	172	14;27,33	+0;0,1
84	13;6,11	-0;0,42	176	14;28,4	+0;0,1
88	13;11,31	-0;0,17	**180**	14;28,35	0

Table 5: The values of a and V_m from ms. Vat., fol. 135v. The bolded entries are common in both Tables 2 and 5.

After computing V_m (in degrees) and e for the moment of sunset, one can find the condition of visibility at the three levels from Table 3 or 4. At a given level, each value of V_m corresponds to two extreme values, e_1 and e_2. The limit e_1 is the least critical value of visibility; namely, if $e < e_1$, according to the text, the moon will never be visible at that level. The upper limit e_2 indicates the value from which the moon will positively be visible in daylight before sunset, therefore if $e > e_2$ there is no need for more computations.[22] Of course, if the moon is closer to the perigee (and hence V_m is larger), the crescent can be more easily sighted and it will be visible for the lower limits of e at all three levels. If the value of e lies between the two extremes at a certain level, the visibility should be checked by further computations for that level.

[22] Ms. BM, is corrupted and the text of the second criterion is unrecoverable, but among the tables there is a table corresponding to the second criterion next to Thābit's one, with similar values compared to Vat.

The modern formula for al-Khāzinī's computational method for each level is as follows:

$$D = e_1 - ((e - e_1).\, e_1 / (e_2 - e_1)) \qquad (6)$$

The moon will be visible only if $de \geq D$.

It is interesting that if $e = e_1$, then, according to formula 6, $D = e_1$, so that the sighting of the crescent is possible only if $de \geq e_1$. Obviously this situation requires that $e_1 \approx al$, which rarely occurs since normally $e_1 > al$, and it causes $e_1 > de$. Therefore, in practice it is almost impossible to observe a crescent near the lower limits in al-Khāzinī's second criterion. If the value of e increases, the critical value of D decreases because the observational condition of the crescent improves, then it reaches zero (i.e., there is no need for the sun to be below the horizon at moonset, so the crescent is visible in daylight).

The validity of formula 6 may be justified by considering the concept of linear interpolation. The linear interpolation method had been known from Babylonian astronomy;[23] Ptolemy applied it in the *Almagest* for different computations.[24]

In accordance with formula 6, I have computed the different values of D at the "general" level, from ms. Vat., with regard to variations of e, e_1, e_2 and V_m. The results are shown in the three-dimensional diagram in Fig. 2. As can be seen from Table 3, at none of the three levels of visibility will the lunar crescent ever be visible in the evening sky even under the best conditions when $e < 8;10°$, and in daylight when $e < 22;25°$. Obviously from Table 4, visibility is completely impossible when $e < 8;50°$ and in daylight when $e < 19;25°$.[25]

[23] See Meijering, pp. 319-342.

[24] Ptolemy, pp. 99, 264. For interpolation methods underlying some of the tables of the *Almagest*, see Van Brummelen, pp. 297-311.

[25] For modern observations, the lunar crescent with 9.95° of angular separation from the sun is the least separation ever observed with the naked eye; moreover, the Council of the Calendar at the University of Tehran, responsible for organizing the formal calendar of Iran, is carrying out a project on the visibility of lunar crescent in daylight with the naked eye and telescopes. On 29 April 2006 (12:50 LT) the lunar crescent with 21.16° of angular separation from the sun was the best record observed with naked eye in daylight (observer's geographical position: λ = 53.57° E and φ =29.22° N) by Mr. M. Sharifi. This shows that the tables of MT and HM probably have a better accordance with modern observations of lunar crescent visibility.

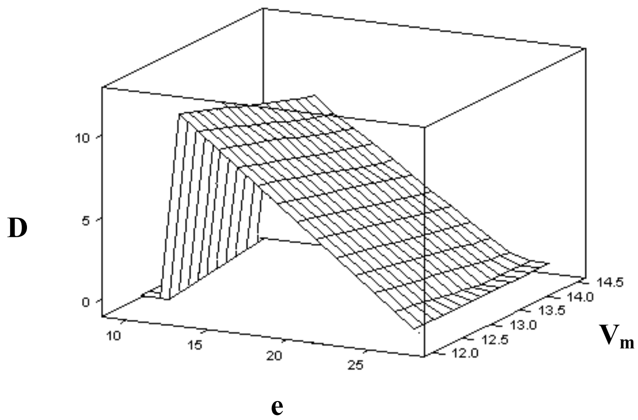

Fig. 2: The three-dimensional diagram with regard to the lunar parameters in the "general" level, on the basis of ms. Vat. The crescent is visible only if lying above the extended surface.

As we have seen, the second criterion deals ambiguously with crescent visibility in daylight. According to al-Khāzinī's description, the crescent is observable in daylight if $e \geq e_2$, but there is no discussion about the exact moment of visibility. However, this part of the criterion needs more investigation with regard to modern naked eye observations, in order to be able to say at what moment in daylight the lunar crescent is observable.

An important question is whether al-Khāzinī was aware of the exact effect of the lunar distance on the visibility of the crescent. The values in Tables 3 and 4 may shed light on this point.

Now we are able to compare the observational condition of different crescents by computing their angular thicknesses (which varies with the lunar distance from the earth) by the following modern formula:[26]

$$TH = SD\,(1 - \cos e) \quad (7)$$

Here SD is the angular radius of the moon (at perigee 0;16,46° and at apogee 0;14,41° corresponding to the respective values of V_m) and TH is the greatest angular thickness of the illuminated part of the crescent. In accordance with

Interestingly Pingree, 1999, pp. 105-113 also supposed that MT and HM might have come down to us from an earlier manuscript of the *Sanjarī Zīj* than BM and Vat.

[26] The formula was concluded from Smart, pp. 166-168.

the above formula, I have computed TH for the extreme values of e_1 from Tables 3 and 4, and the results are shown in Table 6.

V_m	general (easily visible)		moderate (moderately visible)		acute (rarely visible)	
	e_1	TH	e_1	TH	e_1	TH
Ms.Vat.						
12;06º	11;50º	0.312′	10;45º	0.257 ′	9;40º	0.208′
14;27	10;00	0.255	9;05	0.210	8;10	0.170
Ms.MT						
12;06º	11;50º	0.312′	10;40º	0.254′	9;30º	0.201′
14;27	11;10	0.317	10;00	0.255	8;50	0.199

Table 6: The greatest angular thickness of the illuminated part of the crescents with regard to the extreme values of e_1 from mss. Vat., and MT.

As Table 6 shows, in ms. Vat., the angular thicknesses of crescents at different levels decrease from top to bottom, namely the observational condition of crescents declines at each level. But the critical value e_1, at the "general" level, is 10;0º for V_m = 14;27º (i.e., at perigee), preceding the next, moderate level, where e_1 is 10;45º for V_m = 12;6º (i.e., at apogee), both resulting in close angular thicknesses (i.e., 0.255' and 0.257' respectively). This pattern is repeated between "moderate" and "acute" levels as well. Thus it seems that those cases are observationally similar. On the other hand, in ms. MT, the values e_1 lead to similar angular thicknesses at each level, which is as expected. This shows that al-Khāzinī could only observe some crescents with specific values of e_1 and then would reproduce other values of e_1 with equal angular thicknesses to those crescents observed by means of a formula similar to formula 7. Interestingly, extreme values of e_2 in both mss. Vat. and MT separately yield close results of angular thicknesses in their own levels (see Table 7).

Although it seems that the values of MT and HM have a better agreement with modern observations on lunar crescent visibility (see note 25), there are clear gaps regarding the values of each level with the succeeding ones; namely the value 11;10º in the "general" is followed by 10;40º in the "moderate", and the situation of intermediate crescents such as 10;40º < e_1 < 11;10º seems to be unclear. This problem is repeated for e_2. Even though

both Vat. and BM appear to have been copied earlier than MT and HM, it is now impossible to verify the authenticity of each of them, and all of the manuscripts are disordered and incomplete.

V_m	general (easily visible)		moderate (moderately visible)		acute (rarely visible)	
	e_2	*TH*	e_2	*TH*	e_2	*TH*
Ms.Vat.						
12;06°	26;55°	1.591′	25;25°	1.421′	23;55°	1.261′
14;27	25;00	1.571	23;40	1.410	22;25	1.267
Ms.MT						
12;06°	26;00°	1.486′	23;28°	1.214′	20;55°	0.968′
14;27	24;36	1.522	22;01	1.223	19;25	0.954

Table 7: The greatest angular thickness of the illuminated part of the crescents with regard to the extreme values of e_2 from mss. Vat., and MT.

Recalculation of Tables

There is no concrete explanation in al-Khāzinī's *Zīj* that would show which trigonometric functions were used to compute the values of Tables for lunar crescent visibility. The aim of this section is to show a possible procedure which might have been used by al-Khāzinī to calculate his tables, although, obviously, the results obtained are not conclusive. In ms. Vat. fols. 63r-63v there is a rule relating angular diameter of the moon, d_m with V_m (in minutes of arc) as follows:[27]

$$d_m = (V_m \ . \ 392)/9487 \qquad (8)$$

By considering $SD = (d_m)/2$, the above formula may be changed to the following one:

$$SD = 0.02066. \ V_m \qquad (9)$$

[27] This formula was previously used by al-Khwārizmī in his *zīj*. See Neugebauer, pp. 58-59. Of course, he applies 10/247 instead of 392/9487, which yields a close result.

Now, in formula 7, replacing *SD* by (.02066. V_m) leads to:

$$TH = 0.02066.\ V_m\,(1 - \cos e) \qquad (10)$$

For the Vat. Table, at the "general" level, the extreme values of e_1 (=11;50º), V_m (=12;06º) at apogee and e_1 (=10;00º), V_m (=14;27º) at perigee may be applied in formula (10), with results of TH_{max} = 0.3188 and TH_{min} = 0.2721 respectively.

Levels	V_m	e_1	TH_{max}	TH_{min}	Δ	e_2	TH_{max}	TH_{min}	Δ
Ms.Vat.									
general	12;06º 14;27	11;50º 10;00	.3188	 .2721	.0467	26;55º 25;00	 1.6782	1.6249	.0533
moderate	12;06º 14;27	10;45º 9;05	.2632	 .2246	.0386	25;25º 23;40	 1.5065	1.4518	.0547
acute	12;06º 14;27	9;40º 8;10	.2130	 .1816	.0314	23;55º 22;25	 1.3535	1.2879	.0656
Ms. MT									
general	12;06º 14;27	11;50º 11;10	 .3391	.3188	.0203	26;00º 24;36	 1.6258	1.518	.1078
moderate	12;06º 14;27	10;40º 10;00	 .2721	.2592	.0129	23;28º 22;01	 1.3063	1.241	.0653
acute	12;06º 14;27	9;30º 8;50	 .2125	.2057	.0068	20;55º 19;25	 1.0187	.9884	.0303

Table 8: Values used in equations 11-13 for reproducing e_1 and e_2 in mss. Vat and MT.

The value of *TH* varies from top to bottom at the "general" level in this range. The difference between 0.3188 and 0.2721 is Δ = 0.0467. The same calculation may be made for the other columns (see Table 8). By considering variations of *TH* and *f*(*a*), for re-computing e_1, the following functions are proposed:

$$e_1 \text{ Vat.: } (0.02066.V_m\,(1 - \cos e_1)) - (TH_{max} - (\Delta.\,f(a)/60)) = 0 \qquad (11)$$

The calculated values of TH for e_2, unlike e_1, increase from top to bottom for each level. Therefore, to re-compute e_2, the following functions are considered:

$$e_2 \text{ Vat.: } (0.02066.V_m\,(1-\cos e_2)) - (TH_{\min} + (\Delta.f(a)/60)) = 0 \qquad (12)$$

The analysis of Table MT shows that a recomputation of both e_1 and e_2 is possible using the following function:

$$(0.02066.V_m\,(1-\cos e_1)) - (TH_{\min} + (\Delta.f(a)/60)) = 0 \qquad (13)$$

To compute the values of e_2, cos e_1 should be changed to cos e_2 in formula 13.

The results of the computations of e_1 and e_2 obtained by using the aforementioned functions are shown in parentheses in Tables 3 and 4. It seems that Table MT would have been better organized than Table Vat., because the pattern of differences is approximately similar for all columns of e_1. This is as expected, because the values derived from formula 7 also showed a better agreement for MT.

It is also important to establish whether some functions could relate e_1 and e_2 or not. It is clear that each function for e_1, on a certain level, is equivalent to another one for e_2, because both functions are equal to zero. Thus merging the functions 11,12 and 13 ($e_1$and e_2) lead to the following functions for re-computing e_2 from e_1 for each level (for recomputed values see Tables 9 and 10):

Ms. Vat. :

e_2 = arc cos $[1/(0.02066.\ V_m).\ ((-\Delta_{(e_2)} - \Delta_{(e_1)}).\ (f(a)/60) - (TH_{\min\,(e_2)} - TH_{\max\,(e_1)})) + \cos e_1]$

"general": e_2 = arc cos $[\ 1/(0.02066.\ V_m).\ (-(f(a)/600) - 1.3061) + \cos e_1]$ (14)

"moderate": e_2 = arccos $[\ 1/(0.02066.\ V_m).\ (-(f(a)/643\ ) - 1.1886) + \cos e_1]$ (15)

"acute": e_2 = arc cos $[\ 1/(0.02066.\ V_m).\ (-(f(a)/619\ ) - 1.0749) + \cos e_1]$ (16)

Ms. MT :

e_2 = arc cos $[1/(0.02066.\ V_m).\ ((-\Delta_{(e_2)} + \Delta_{(e_1)}).\ (f(a)/60) - (TH_{\min\,(e_2)} - TH_{\min\,(e_1)})) + \cos e_1]$

"general": e_2 = arc cos $[\ 1/(0.02066.\ V_m).\ (-(f(a)/686) - 1.1992) + \cos e_1]$ (17)

"moderate": e_2 = arccos$[1/(0.02066.V_m).(-(f(a)/1145) - 0.9818) + \cos e_1]$ (18)

"acute": e_2= arccos$[1/(0.02066.V_m).(-(f(a)/2553) - 0.7827) + \cos e_1]$ (19)

Interestingly, using formula 7 and comparing the ratio of angular thickness of crescents between e_1 and e_2 in different levels leads to a very approximate function for reproducing the values e_2 from e_1 for the Table MT in all of these levels:

$$TH\,(e_2) \approx 4.8\ TH\,(e_1)$$

$$SD\,(1-\cos e_2) = 4.8\ SD\,(1-\cos e_1)$$

$$e_2 = \arc\cos\,((4.8 \cos e_1) - 3.8) \tag{20}$$

The results reproduced by the above simple equation (20) are shown in Table 11. This equation raises the hypothesis that the Table MT might have been computed using a single equation such as 20, rather than a group of equations.

Concluding remarks

In his *Zīj*, al-Khāzinī first explains the critical values of lunar crescent visibility provided by his Muslim predecessors, with whom I have not dealt in this article. This section is historically very important because al-Khāzinī tried to present a complete sketch of the achievements up to his time, based on what he knew and on the importance of the methods. The text and structure of the *Sanjarī Zīj* shows that al-Khāzinī presents his first criterion as the culmination of a group of criteria (based either on one or on several arcs) that were originally presented by al-Khwārizmī and, in the same century, by Ḥabash al-Ḥāsib and, later, by Kūshyār Gīlānī[28] (according to al-Khāzinī). Of course, al-Khāzinī also considers the effect of the lunar distance on the above arcs, which was ignored by the astronomers mentioned above.

The text continues with the details of another group of criteria used by al-Battānī and Thābit ibn Qurra (Chapters 4 and 5 of Essay 9: the chapter on al-Battānī has not been edited).[29] These criteria consist of computational procedures and they are appended by separate tables. Al-Khāzinī ends this chapter with his second, more sophisticated criterion, and we can consider it as the culmination of the second group. The tabular

[28] For Kūshyār's criterion, see Bagheri, pp. 57-58 (for its Arabic text), pp. 83-84 (English translation) and p. 91 (commentary).
[29] Morelon, pp. 114-115, has edited the text of Thabit's criterion.

part of al-Khāzinī's second criterion appears next to Thābit's table. These two criteria have basic similarities, because both apply the lunar distance from the earth and distinguish two extremes for lunar crescent visibility.[30] Undoubtedly al-Khāzinī presented this criterion under the influence of Thābit ibn Qurra, but he endeavored to construct a new criterion showing, furthermore, the effect of human and atmospheric conditions on lunar crescent visibility.[31] Al-Khāzinī distinguishes three levels of visibility (general, moderate and acute), and thus introduces a concept of frequency in crescent visibility. Since we may assume that the introduction of these three levels was based on the study of successful and unsuccessful sightings of the crescent over a long observational period, it may be considered as a first step on the way to achieving a notion of "probability" in lunar crescent visibility. Unfortunately, we have no information either on al-Khāzinī's observations or on the procedure that led him to his second criterion. In any case, judging from the investigations carried out to date, this criterion is unprecedented in the history of astronomy before al-Khāzinī.

Overall, we can conclude that al-Khāzinī's first criterion was apparently intended for approximate, rapid determinations of lunar crescent visibility, and the second was developed for exact judgments. As far as I know, in later centuries only the first criterion of al-Khāzinī was applied by an Indian (?) astronomer, Maḥmūd ibn 'Umar in his *Zīj-e Nāṣerī* (compiled in Persian in Delhi, ca. 643 A.H/1245A.D.) with some small modifications.[32] An important historical question is why al-Khāzinī's criteria did not gain prevalence in later *zīj*es, while in the next century a simpler and less exact criterion proposed by Naṣīr al-Dīn al-Ṭūsī

[30] See ms. Vat., fol. 143r, and note 6.

[31] According to al-Bīrūnī's citation in the *al-Qānūn*, p. 945, the famous Persian astronomer al-Nayrīzī (d. 310 A.H./ 922 A.D.), was probably the first to discuss in general terms the effect of the lunar distance from the earth and the effect of atmospheric conditions, in different seasons of the year, on lunar crescent visibility.

[32] In the *Zīj-e Nāṣerī,* Mar'ashi library (Qom), ms. No. 9176, fol. 2r, Maḥmūd ibn 'Umar mentions the names of several astronomers including al-Khāzinī who inspired him in composing his *zīj*. The table of mean motions of planets in the *Zīj-e Nāṣerī* (fol. 135v) is dated 615 Yazdgerdi (643 A.H/1245 A.D.) and the author dedicated his work to Nāṣer al-Dīn Maḥmūd (reign: 644-664A.H/ 1246-1265A.D.), the seventh sultan of the *shamsiyya* sultans in India. See also fol. 101v. for two tables on lunar crescent visibility. The lower table yields values of al-Khāzinī's first criterion which are very similar to those of BM. For further explanation of the *Zīj-e Nāṣerī*, see Van Dalen, 2004, pp. 825-862.

(597-672 A.H./1201-1274 A.D.) was included in the *Īlkhānī Zīj* and gained wide acceptance and was frequently used by later Islamic astronomers.[33] In answering this question, both scientific and social developments should be considered. From the scientific viewpoint, Muslim astronomers found both of al-Khāzinī's criteria difficult to use, especially the second one. It was not as easy for them to compute the different mentioned arcs and V_m as it is today. On the other hand, the second criterion would have been confusing for astronomers and, especially for those persons with a modest knowledge of astronomy who organized the civil lunar calendar in that period.[34] They probably preferred to use a simple though inexact criterion with straightforward extremes rather than a complex one based on limits without a conclusive result.

From the social point of view, the outrage of the Mongol attack on central Asia and Iran heavily disrupted the diffusion of earlier astronomical traditions and one finds few traces of earlier criteria for lunar crescent visibility (including al-Khāzinī's criteria) in the *zīj*es of the Mongol period or, consequently, in later centuries. Moreover, the prevalence of astronomical methods (including lunar crescent visibility theories) was quite possible under the influence of schools as promoters of certain scientific methods in different periods. In the case of al-Khāzinī, no astronomical school has yet been associated with him, whereas Naṣīr al-Dīn al-Ṭūsī and his colleagues at the Maragha observatory made up a research circle that had a profound effect on later astronomers. Needless to say, the political role of al-Ṭūsī also increased the fame of that circle.[35] This is confirmed by the numerous manuscripts that have survived of the *Īlkhānī Zīj*, whereas of the *Sanjarī Zīj* only a few remain.[36] This latter subject is related to the social and cultural aspects of science in the

[33] The criterion of al-Ṭūsī was simply based on $\Delta\lambda$ and L. See Giahi Yazdi, pp. 231-243.

[34] In Vat., fol. 143r (see plate in appendix) and in HM, fol. 31v, there is a warning to the effect that the lunar calendar should be organized on the basis of the lunar visibility <up to> the "moderate" level nor "acute" level, and if there is a possibility in "acute" level, it should only be marked in the margin of calendar.

[35] For general characteristics of the Maragha school, see Saliba, 1987, pp.361-373 and 1991, pp. 67-99 and for the Maragha observatory and its relation with al-Ṭūsī see, Sayili , pp. 189-223.

[36] For the list of manuscripts of the *Īlkhānī Zīj* only in the libraries of Iran see Monzavi, p. 300, whereas only four manuscripts have survived from the *Sanjarī Zīj*, see note 3.

Islamic Middle Ages, which have not been thoroughly investigated to date.

sixtieths $f(a)$	general (easily visible)		moderate (moderately visible)		acute (rarely visible)	
	e_1	e_2	e_1	e_2	e_1	e_2
0' 0"	11;50°	26;55°(0)	10;45°	25;25°(0)	9;40°	23;55°(0)
0 21	11;49	26;55(+1)	10;45	25;25(+1)	9;40	23;55 (+1)
0 42	11;48	26;54(+2)	10;44	25;24(+1)	9;39	23;54 (+1)
1 42	11;47	26;52(+2)	10;42	25;22(+2)	9;38	23;53 (+2)
2 42	11;45	26;50(+3)	10;40	25;20(+3)	9;37	23;52 (+3)
4 01	11;42	26;47(+3)	10;38	25;18(+3)	9;34	23;49(+3)
5 21	11;40	26;44(+3)	10;36	25;15(+3)	9;32	23;47(+4)
7 18	11;37	26;41(+4)	10;33	25;12(+3)	9;30	23;45(+4)
9 15	11;34	26;36(+2)	10;29	25;08(+3)	9;27	23;42(+4)
11 37	11;30	26;31 (0)	10;25	25;05(+3)	9;23	23;39(+3)
14 00	11;25	26;26(-1)	10;21	25;01(+2)	9;19	23;34(+1)
16 48	11;20	26;23(-1)	10;16	24;58(+2)	9;15	23;31(0)
19 36	11;15	26;19(0)	10;11	24;53(+1)	9;10	23;28(+1)
22 36	11;10	26;14(0)	10;05	24;48(+2)	9;06	23;22(-1)
25 36	11;04	26;09(+2)	10;00	24;42(+2)	9;01	23;18(+1)
28 42	10;58	26;03(+3)	9;56	24;36(+2)	8;57	23;14(+2)
31 48	10;53	25;58(+4)	9;52	24;30(+1)	8;52	23;10(+3)
34 54	10;47	25;52(+3)	9;47	24;26(+2)	8;48	23;05(+2)
38 00	10;41	25;47(+3)	9;42	24;22(+2)	8;43	23;01(+1)
41 00	10;35	25;42(+4)	9;37	24;18(+3)	8;39	22;58(+3)
44 00	10;29	25;37(+4)	9;32	24;14(+3)	8;34	22;54(+2)
46 45	10;24	25;32(+4)	9;27	24;09(+3)	8;30	22;49(+2)
49 30	10;19	25;27(+4)	9;24	24;05(+3)	8;26	22;46(+2)
51 39	10;15	25;22(+3)	9;20	24;02(+4)	8;22	22;42(+1)
53 48	10;12	25;17(0)	9;16	23;58(+2)	8;19	22;38(0)
55 32	10;09	25;13(-1)	9;12	23;53(+1)	8;16	22;35(-1)
57 15	10;08	25;10(-2)	9;10	23;50(0)	8;14	22;32(-2)
58 18	10;05	25;07(-1)	9;08	23;47(0)	8;13	22;30(-2)
59 21	10;04	25;05(-1)	9;07	23;44(-1)	8;12	22;28(-1)
59 41	10;02	25;02(0)	9;06	23;42(0)	8;11	22;26(0)
60 00	10;00	25;00(0)	9;05	23;40(0)	8;10	22;25(0)

Table 9: The recomputed values of e_2 based on e_1 and $f(a)$ for ms. Vat.

sixtieths $f(a)$	general (easily visible)		moderate (moderately visible)		Acute (rarely visible)	
	e_1	e_2	e_1	e_2	e_1	e_2
0' 0''	11;50º	26;00º(0)	10;40º	23;28º(0)	9;30º	20;55º(0)
0 21	11;50	26;00(+1)	10;40	23;28 (0)	9;30	20;54(0)
0 42	11;50	25;59(+1)	10;40	23;27(+1)	9;30	20;53(0)
1 42	11;49	25;58(+2)	10;39	23;26(+2)	9;29	20;52(+1)
2 42	11;48	25;57(+4)	10;38	23;24(+2)	9;28	20;50(+1)
4 01	11;47	25;55(+4)	10;37	23;22(+2)	9;27	20;49(+3)
5 21	11;46	25;53(+4)	10;36	23;20(+3)	9;26	20;47(+3)
7 18	11;45	25;50(+4)	10;35	23;18(+4)	9;25	20;45(+4)
9 15	11;44	25;47(+3)	10;34	23;15(+3)	9;24	20;42(+4)
11 37	11;42	25;44(+3)	10;32	23;11(+2)	9;22	20;38(+3)
14 00	11;40	25;40(+3)	10;30	23;07(+1)	9;20	20;34(+2)
16 48	11;38	25;36(-1)	10;28	23;03(-1)	9;18	20;29(-1)
19 36	11;36	25;32(-2)	10;26	22;59(-2)	9;16	20;25(-1)
22 36	11;35	25;28(-2)	10;25	22;54(-3)	9;15	20;20(-2)
25 36	11;33	25;24(-1)	10;23	22;49(-2)	9;13	20;16(-1)
28 42	11;31	25;20(0)	10;21	22;44(-2)	9;11	20;12(0)
31 48	11;29	25;16(+1)	10;19	22;39(-2)	9;09	20;07(0)
34 54	11;27	25;12(0)	10;16	22;35(-2)	9;06	20;03(+1)
38 00	11;25	25;07(-2)	10;14	22;31(-3)	9;04	19;58(0)
41 00	11;22	25;02(-2)	10;12	22;28(-2)	9;02	19;54(0)
44 00	11;20	24;58(-3)	10;10	22;23(-3)	9;00	19;49(-1)
46 45	11;18	24;54(-3)	10;08	22;19(-3)	8;58	19;45(-1)
49 30	11;16	24;51(-2)	10;07	22;15(-4)	8;57	19;41(-2)
51 39	11;15	24;48(-3)	10;05	22;11(-5)	8;55	19;38(-2)
53 48	11;14	24;45(-4)	10;04	22;07(-7)	8;54	19;34(-3)
55 32	11;13	24;42(-5)	10;03	22;04(-8)	8;53	19;31(-4)
57 15	11;12	24;40(-5)	10;02	22;03(-6)	8;52	19;29(-4)
58 18	11;11	24;39(-3)	10;01	22;02(-5)	8;51	19;27(-3)
59 21	11;11	24;38(-2)	10;01	22;01(-4)	8;51	19;26(-3)
59 41	11;10	24;37(0)	10;01	22;01(-2)	8;51	19;25(-1)
60 00	11;10	24;36(0)	10;00	22;01(0)	8;50	19;25(0)

Table 10: The recomputed values of e_2 based on e_1 and $f(a)$ for ms. MT.

general (easily visible)		moderate (moderately visible)		acute (rarely visible)	
e_1	e_2	e_1	e_2	e_1	e_2
11;50º	26;00º(-6)	10;40º	23;28º(-2)	9;30º	20;55º(0)
11;50	26;00(-6)	10;40	23;28(-2)	9;30	20;54(0)
11;50	25;59(-7)	10;40	23;27(-3)	9;30	20;53(-1)
11;49	25;58(-6)	10;39	23;26(-2)	9;29	20;52(0)
11;48	25;57(-5)	10;38	23;24(-2)	9;28	20;50(0)
11;47	25;55(-5)	10;37	23;22(-1)	9;27	20;49(+1)
11;46	25;53(-4)	10;36	23;20(-1)	9;26	20;47(+1)
11;45	25;50(-5)	10;35	23;18(-1)	9;25	20;45(+2)
11;44	25;47(-6)	10;34	23;15(-2)	9;24	20;42(+1)
11;42	25;44(-4)	10;32	23;11(-1)	9;22	20;38(+1)
11;40	25;40(-4)	10;30	23;07(-1)	9;20	20;34(+2)
11;38	25;36(-3)	10;28	23;03(0)	9;18	20;29(+1)
11;36	25;32(-3)	10;26	22;59(0)	9;16	20;25(+2)
11;35	25;28(-5)	10;25	22;54(-3)	9;15	20;20(-1)
11;33	25;24(-4)	10;23	22;49(-3)	9;13	20;16(0)
11;31	25;20(-4)	10;21	22;44(-4)	9;11	20;12(0)
11;29	25;16(-3)	10;19	22;39(-4)	9;09	20;07(-1)
11;27	25;12(-3)	10;16	22;35(-2)	9;06	20;03(+2)
11;25	25;07(-3)	10;14	22;31(-1)	9;04	19;58(+1)
11;22	25;02(-2)	10;12	22;28(0)	9;02	19;54(+2)
11;20	24;58(-1)	10;10	22;23(0)	9;00	19;49(+1)
11;18	24;54(-1)	10;08	22;19(0)	8;58	19;45(+1)
11;16	24;51(+1)	10;07	22;15(-2)	8;57	19;41(0)
11;15	24;48(0)	10;05	22;11(-1)	8;55	19;38(+1)
11;14	24;45(-1)	10;04	22;07(-3)	8;54	19;34(0)
11;13	24;42(-2)	10;03	22;04(-4)	8;53	19;31(-1)
11;12	24;40(-1)	10;02	22;03(-2)	8;52	19;29(-1)
11;11	24;39(0)	10;01	22;02(-1)	8;51	19;27(-1)
11;11	24;38(-1)	10;01	22;01(-2)	8;51	19;26(-2)
11;10	24;37(0)	10;01	22;01(-2)	8;51	19;25(-3)
11;10	24;36(-1)	10;00	22;01(0)	8;50	19;25(-1)

Table 11: The recomputed values of e_2 based on e_1 for ms. MT (simple equation).

Acknowledgements

My research was partly financed by the University of Tehran (through the Institute for the History of Science, project no.300/2/763). This support is gratefully acknowledged. It is a pleasure to record my gratitude to Prof. Samso, Prof. Djebbar, Dr. Van Dalen, Dr. Hogendijk, and Dr. Bagheri for their comments on this article. I am also grateful to Dr. Salmani for some helpful hints in the translation of the Arabic text.

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Appendix

The text of al-Khāzinī’s criteria from ms. Vatican, Arab, 761. The manuscript errors are corrected in footnotes and some Arabic words have been rewritten according to modern spelling. My explanatory additions to the text are provided in brackets [ ].

الباب الثاني: في ذكر من اعتمد على القسي المفردة البسيطة

القسي المفردة المستعملة في عمل معرفة رؤية الهلال اربع[1]. احداها قوس النور والثانية قوس المكث والثالثة قوس الانحطاط والرابعة قوس الارتفاع. فصل قالت طائفة من اهل الكتاب: من عشر[2] الى اثنتي عشرة فما فوقها فالقمرفي حدّ الرؤية فيمكن ان يرى. فصل قالت [طائفة من] الهند والسند[3] و محمّد بن موسى الخوارزمى: اذا كانت قوس المكث اثنتي

عشرة فما فوقها يرى الهلال والاّ فلا. فصل وقال احمد بن عبد الله حبش الحاسب: اذا كانت[4] انحطاط الشمس عند مغيب القمر عشرة اجزاء فما فوقها يرى و الاّ فلا. و روي السعيدي عنه انّها اذا كانت[5] [انحطاط الشمس] ثمانية اجزاء و ثلثي جزء فما فوقها يرى و ان كانت[6] اقلّ منها فلا. فصل وقال كوشيار: المعتبر عندي ان تكون قوس النور عشرة اجزاء و قوس المكث ثمانية اجزاء و قوس الارتفاع المرئي ستـّة اجزاء و قوس الانحطاط [من] ثمانية الى سبعة اجزاء و نصف، فان كانت هذه القسي في حدودها [أو] اكثر فالهلال فى حدّ الرؤية يمكن ان يرى و ان كانت اقلّ فلا. و قال ان كان مجموع قوس النوروالانحطاط ثمانية عشر فما فوقها يرى وان شهد اثنان منها فيحكم على الرؤية.

الباب الثالث: فى تصحيح القسي الاربع[7] المفردة للخازنى

القسي الاربع[8] المذكورة يجب ان تصحّح بحسب كون القمر فى فلك التدوير اذ المقدّرة منها والقمر في الذروة اعظم من التي تقدّر و القمر في الحضيض. فالاولى منها قوس النور وهي فيما بين عشرة اجزاء الى اثني عشر. والثانية، قوس المكث وهي فيما بين سبعة اجزاء و نصف الى تسعة اجزاء و الرابعة قوس الارتفاع المرئي و هي فيما بين ستـّة اجزاء و ثلثي جزء الى ثمانية اجزاء بالتقريب. فصل و اذا اردنا تعديلها اخذنا ما بازاء خاصة القمر من دقائق التقويم في جدول الكسوفات و ضربناها في تفاضل كلّ واحدة منها و قسمنا المبلغ على ستـّين، فما حصل نقصناه من اعظمها فتبقى القوس المطلوبة. وان شئنا عدّلناها ببهت القمر ووجهه ان نجعلها دقائق و نقصنا منها 727، فتبقى الفضلة، ضربناها فى التفا ضل و قسمنا المبلغ على 138 فنقصنا الخارج من القسمة من اعظمها، تبقى القوس المطلوبة. حفظنا كلّ واحدة من هذه على حدّه. فصل في رؤية الهلال منها : رجعنا الى القسي الخارجة ليلة الثلاثين من موضع النيـّرين و قابلناها بالمحفوظة. فان كانت مثلها او اكثر، فالهلال في حدّ

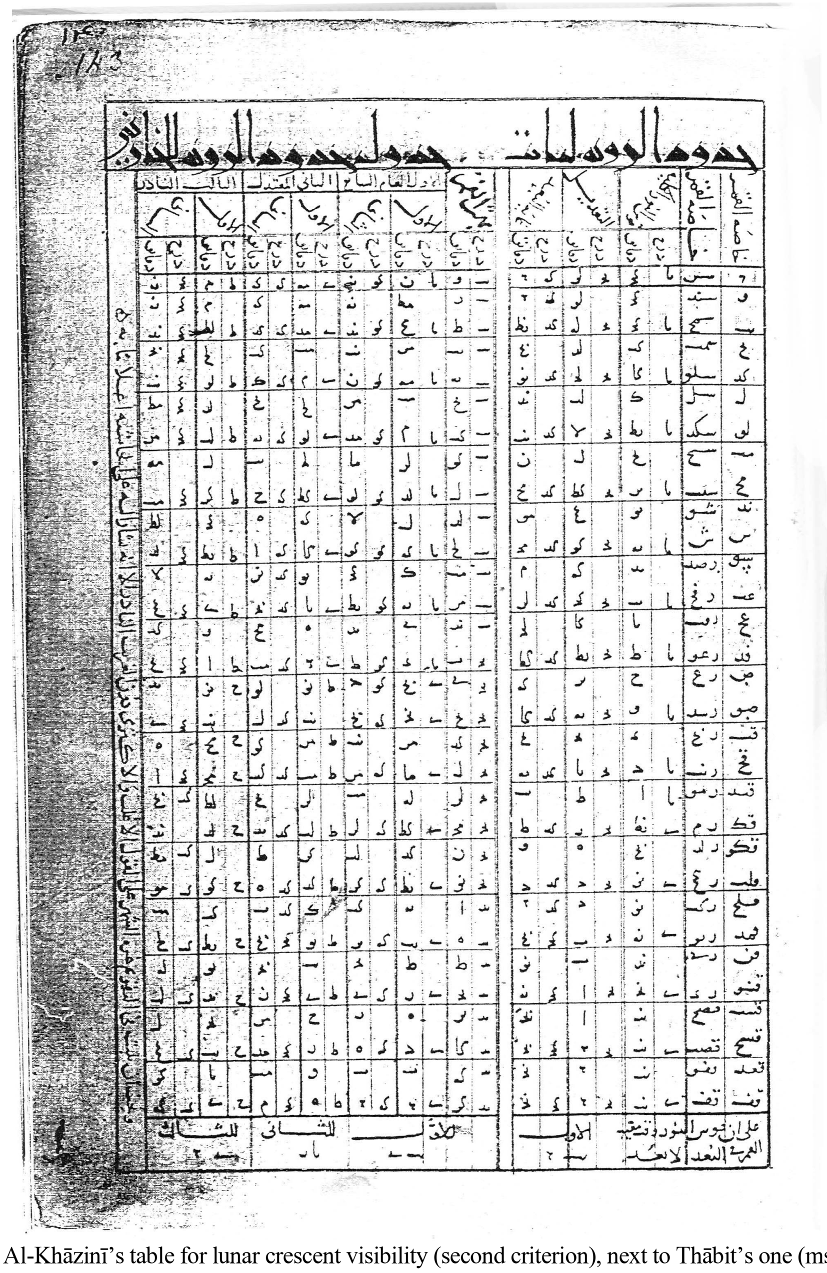
الرؤية يمكن ان يرى وان كانت اقلّ فلا يرى و ان دلـّت بعضها على الرؤية فالحكم فيهما للغلبة والاعتماد على المركـّب من قوس النور والانحطاط دون سائرها فى هذا الباب لأن قوس النور اذ زادت على اربعة و عشرين يرى الهلال قبل مغيب الشمس وان لم تدلّ قسي المكث والانحطاط والارتفاع في بعض الاوضاع [و] المنطقة والبقاع على الرؤية .

الباب السادس: في ذكر ما اعتبره الخازني في الرؤية

وهذا الباب يتضمّن تعديل انحطاط الشمس الكلـّي بحسب قوس النور و يسمّى بعد تعديله قوس الرؤية المصحّحة. نقول قوّمنا النيّرين ليلة الثلاثين من الشهر العربي وقت مغيب القمر و عرفنا من تقويميهما المصحّحين، كما ذكر قبل، قوسي النور و الانحطاط الموجودتين و حفظناهما. فصل فى معرفة الحدود الكلـّية من الجدول: دخلنا بخاصّة القمر فى احد الجداول الثلاثة من حدود الرؤية الموقع عليه الخازني الذي هو اليق للفصل[9] والبلد و المتأملين حسب ما وقعت التجربة به و اخذنا ما بحيالها من قوسين حدّي الاوّل و الثاني وحفظنا كلّ واحدة منهما على حدّه. ثم نقصنا الحدّ الاوّل من الحدّ الثاني و سمّينا الباقي قوس التعديل و حفظناها ايضا ناحية. فصل فى الاعتبار: اوّلا نظرنا الى قوس النور الموجودة فان كانت اقلّ من قوس الحدّ الاوّل فلا يطمع في رؤية الهلال لانـّه تحت الشعاع بعد فلم يخرج الى امكان حدّ الرؤية، و ان كان مثل الحدّ الثاني او اكثر[منه] فقد برز عن الشعاع فيمكن ان يرى نهارا قبل مغيب الشمس فلا نحتاج الى التأمـّل فيه. فصل في تصحيح قوس الرؤية : اذا كانت قوس الرؤية اكثر من الحدّ الاوّل و اقلّ من الثاني فالهلال في حدّ الاشتباه وعلى طرفي وجوب الرؤية و امتناعها فنحتاج الى فضل عناية و عمل. ووجهه ان نستخرج اوّلا قوس الرؤية الكلـّية بحسب الابعاد بان ننقص الحدّ الاوّل الذي خرج من الجدول من قوس النور الموجودة فتبقى الفضلة، ضربناها في الحدّ الاوّل و قسمنا المبلغ على التعديل. فما خرج من

القسمة، نقصناه ابدا من الحدّ الاوّل تبقى قوس الرؤية. و ان شئنا نقصنا الفضلة من التعديل المحفوظ تبقى حصّة قوس الرؤية فنضربها فى الحدّ الاوّل ونقسم المبلغ على التعديل فيخرج قوس الرؤية الكلـّية. فصل في الاعتبار ثانيا[10] رجعنا الى انحطاط الشمس ، فان كان مثل قوس الرؤية الكلـّية او اكثر[منها] فانّ الهلال يرى فى تلك البقعة و ان كان اقلّ منها فلا يرى. فهذا هو الاساس لبقعة خاصة اذا ثبت بالتجربة في واحد من هذه الثلاثة. فصل فى عمل كلـّي وقانون البقاع كلـّها : اذا اردنا ذلك عملنا بكلّ واحد من الجداول الثلاثة بالمؤامرة المذكورة ثمّ بعد الفراغ منها اذا دلّ العمل على الرؤية قلنا في الاوّل تكون الرؤية شائعة و من الثاني دون الاوّل تراه الابصار المعتدلة و الحادّة ان كان الهواء صافيا فلا تكون شائعة. ومن الجدول الثالث دون الاوّلين تراه الابصارالحادّة نادرا ان كان الهواء صافيا واذا لم يدلّ الثالث فلا يرى بالاتفاق في شيء من البقاع لقلـّة النور في جرمه.

[1] اربعة في م.
[2] عشرة في م.
[3] الصين في م.
[4] كان في م.
[5] كان في م.
[6] كان في م.
[7] الاربعة في م.
[8] الاربعة في م.
[9] الفضل في م.
[10] ثان في م.



Al-Khāzinī's table for lunar crescent visibility (second criterion), next to Thābit's one (ms. Vat. fol. 143r).